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THIS JOURNAL IS DEDICATED TO THE FOLLOWING AIMS: (1) Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values. (2) To supply an additional medium for the publication of expository mathematical articles. (3) To promote more scientific methods of teaching mathematics. (4) To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Every paper on technical mathematics offered for publication should be submitted (with enough enclosed postage to cover two two-way transmissions) to the Chairman of the appropriate Committee, or to a Committee member whom the Chairman may designate to examine it, after being requested to do so by the writer.

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Professor James McGiffert

There are some men of whom it can be said without exaggeration, "Where he sits is the head of the table". Such a man was James McGiffert, mathematician, teacher, author, and lecturer. He could talk fluently and with authority on many subjects including his beloved mathematics, astronomy, the classics, Latin and Greek. He was master of the English language and took great pride in The Society of Spoken English which he founded and headed for many years. He astounded laymen and clergymen alike by his knowledge of the Bible and Biblical passages. His associates soon learned to accept his opinion on such matters without question. New acquaintances were impressed immediately by his friendliness, poise, and dignity, and soon came under his magic spell. His memory was phenomenal. He loved to attend the Annual Alumni Meeting at Commencement time, and it was not unusual for him to recognize a former student whom he had not seen in many years merely by the sound of his voice. He could perform mentally involved calculations requiring the use of trigonometric functions and logarithms faster than his students could perform them with the use of tables.

His circle of intimate friends included many nationally known educators and clergymen and many nationally known figures in the business and financial world. The frequent formal dinner parties at which he and Mrs. McGiffert entertained such friends were

marked by excellent food, sparkling conversation, and the inexhaustible supply of anecdotes provided by the host. The dessert was made invariably by Professor McGiffert and consisted of his famous "Modulus" ice cream, which, as he explained, was made according to his special formula. Its quality and consistency were maintained by careful adherence to his "Modulus" rule for determining the number of times the crank should be turned.

Of Scotch-Irish descent, Professor McGiffert was born at Stockport, New York, and received his early training in nearby Hudson where he was graduated from high school at the age of 16. He entered Rensselaer Polytechnic Institute in the fall of 1897 and earned the degree of Civil Engineer four years later. He then spent one year at Johns Hopkins University following which he returned to Troy to accept a position in the Department of Mathematics. Except for leave to study at Harvard where he received his B. A. and M. A. degrees and at Columbia where he earned his Ph.D., he remained at Rensselaer, finally becoming full professor in charge of graduate courses in mathematics. His success as a teacher, both at the Institute and at his home where he taught countless private pupils, was due in a large measure to his great love for his favorite subject which he explained with enthusiam and a lively flow of good English that could not fail to impress even the dullest students.

In his early years as a teacher he published, mainly for the use of his own students, "Notes on Algebra", "Problems on Mensuration," and "Mathematical Shortcuts." In later years he published two texts, "Plane and Solid Analytic Geometry" and "Higher Algebra." Shortly before his death he assisted in the

preparation of a brief introduction to algebra and geometry for use in defense courses.

Rensselaer Polytechnic Institute and his many friends have suffered a great loss indeed in the passing of Professor McGiffert; but those of us who knew him best feel that his brilliance, wisdom, friendliness, and courage influenced so many lives that much of his spirit still lives on.

Rensselaer Polytechnic Institute. **LYNN L. MERRILL.**

BEAUTIFUL LOYALTY!

On December 21, 1943, at the opening of the L. S. U. Christmas holiday period the Editor and Manager was invited to the coffee room in Nicholson Building of the University and presented with a cash sum of $115.50 to be used in furthering the cause of the MAGAZINE.

Under the leadership of Dr. Irby C. Nichols a long-time member of the L. S. U. Mathematics staff, with the encouragement and approval of W. Vann Parker, head of the Department, cash donations had been gathered from exactly 100% of the membership of the mathematics staff. A remarkable testimonial to their high valuation of NATIONAL MATHEMATICS MAGAZINE!

The following is a list of all contributors to the fund and also a complete roster of the L. S. U. mathematics faculty. The donations ranged all the way from $10.00 to $2.00: Lieut.-Col. Perry Cole, J. L. Dorroh, Elizabeth Freas, Yvonne Jones, Houston Karnes, R. C. Murray, Irby C. Nichols, K. L. Nielsen, R. L. O'Quinn, Christine S. Parker, W. V. Parker, Joseph E. Pryor, Ruth E. Ramsey, F. A. Rickey, Carolyn Rosenthal, N. E. Rutt, H. L. Smith, J. C. Stewart, Lurline Sttwart, C. J. Thorne, Marelena White, Paul A. White, Howard Nolan Wright.

Humanism and History of Mathematics

Edited by
G. Waldo Dunnington and A. W. Richeson

John Napier and His Logarithms

By E. R. Sleight
Albion College, Albion, Michigan

John Napier, the inventor of logarithms, was born at Mercheston Castle, now in the city of Edinburgh. At the time of his birth the old castle with its estates was quite outside the city limits. Of his boyhood and early training very little is known. He attended St. Andrews University, entering that institution in 1563. There is no record as to the time spent as a student in St. Andrews. It is certain that he did not acquire his wide knowledge of classical literature, nor was he set upon the path which lead to his invention while a student at St. Andrews. But he did receive an impetus to theological studies which was a lasting benefit and pleasure to him. Theology bulked large in the 16th century discussions, and Napier took his part in these discussions. Perhaps he is as well known by his publication "A plain discovery of the whole revelation of John" as by his logarithms.

During Napier's lifetime Scotland was very unsettled, so his work is all the more remarkable. Edinburgh University was established in 1582, but this does not mean that education flourished, even though this made the fourth institution of its kind in Scotland. Before Napier's time no Scotsman had made an outstanding contribution to scientific learning, and his invention has been the source of great wonder, both to historians and to scientific men. Historians often refer to Buchanan as the outstanding man of this period. He was a great man but not a scientist, so that his greatness in no way detracts from Napier's.

There is evidence that mathematics was of great interest to Napier, even at a very early age as is shown in "De Arte Logistica" published in 1839. In this it appears that he was lead to a consideration of imaginary roots of equations and a general method for extracting roots of all degrees. It must be remembered, however, that he was dealing with subjects far in advance of his age, and also during a

period of no very well defined coordinate geometry, no calculus as we know it,—in fact the idea of index numbers did not exist. His notion of a logarithm involved a perfectly well defined functional relation far in advance of his time. The difficulties of computation at that time caused him to give up his algebraic investigations and turn his attention to labor saving devices. Of these his system of logarithms is the most outstanding, especially as a very distinct mathematical advancement. But for more than one hundred years the common people remembered him more distinctly as the inventor of Neper's* Bones, or Rods, consisting of 10 oblong pieces of wood or other material, with square ends. Each of the four faces of each rod contains multiples of one of the nine digits, the digit itself being placed at the end of the rod. The following table shows the numbering of the rods, as well as the digits appearing at the ends:

TABLE A

NUMBER OF ROD	DIGETS AT TOP	DIGETS AT BOTTOM
1	0,1	9,8
2	0,2	9,7
3	0,3	9,6
4	0,4	9,5
5	1,2	8,7
6	1,3	8,6
7	1,4	8,5
8	2,3	7,6
9	2,4	7,5
10	3,4	6,5

It is to be observed that each rod contains on two of its faces multiples of digits which are complements (sum equals nine) of those on the other two faces, and these complements are reversed on the rod, one being at the end of one face while its complement is at the other end of another. Thus in the rod numbered 8, the 2 and the 7 are on opposite ends, as well as the 3 and the 6. These rods were used very extensively

*This is the spelling used by the common people. Napier spelled his name in four different ways, and I have seen at least seven different forms. In those days very little attention was paid to the spelling of a name.

in Scotland for more than a hundred years in any operation that involved multiplication. They may be used for division, but the process is much more difficult. An example: Suppose it is desired to multiply 2085 by 736. Four rods are selected on each of which appears at the top, one of the digits 0,2,5,8. To these rods is added that one on which unity appears in a similar position. This latter rod is used in all examples in multiplication. These are then placed as indicated in Table B, the diagonal line forming a part of the picture.

To perform the multiplication, the numbers appearing diagonally in row 6 are added. From left to right, these additions yield the following results: 1,2,4,11,0. These are partial products, and their sum, if due regard to position* is taken, gives the product of 6×2085. Similarly by using rows 3 and 7, the results of multiplying 2085 by these digits are obtained. Thus we have for these various partial products,

$$\begin{aligned} 6\times 2085 &= 12510 \\ 3\times 2085 &= \ \ 6255 \\ 7\times 2085 &= 14595 \end{aligned}$$

If now these products be placed in position according to the usual process, and the sum be taken we have for the final result

$$\begin{array}{r} 12510 \\ 6255 \\ 14595 \\ \hline 1534560 \end{array}$$

While the common people remember Napier for these mechanical devices, the scientific world thinks of him as the inventor of logarithms, and there are many evidences of the fact that this invention was immediately and universally appreciated. He presented his system to the world in 1614 in what he chose to call his Descriptio. In this he did not give a full account of the method used, stating that he "preferred to await the judgment and censur of learned men." The Descriptio was written in Latin. It contained only 57 pages of descriptive matter, but in it are found 90 pages of tables. The descriptive

*Placed in position these numbers are:

$$\begin{array}{r} 0 \\ 11 \\ 4 \\ 2 \\ 1 \\ \hline 12510 \end{array}$$

matter contains an account of Napier's idea of a logarithm, together with some of the properties. He also applied them to plane and spherical triangles. His rules for circular parts as applied to right angled triangles form a part of this treatise.

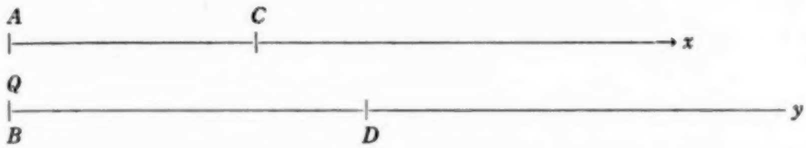
TABLE B

2	0	8	5	1
/4	0/0	1/6	1/0	2
/6	0/0	2/4	1/5	3
/8	0/0	3/2	2/0	4
1/0	0/0	4/0	2/5	5
1/2	0/0	4/8	3/0	6
1/4	0/0	5/6	3/5	7
1/6	0/0	6/4	4/0	8
1/8	0/0	7/2	4/5	9

His untimely death in 1617 prevented the fulfillment of his desire to present his plan in full. In 1619 his son Robert in the Constructio supplied the necessary detail. The first two pages are in the nature of a preface in which Robert extols the virtues of his father. Then follows 57 pages in which the conception of a logarithm is clearly defined, and the successive steps in their development are fully detailed. In this work one of Napier's four analogies is given, the other three being worked out later by Briggs.

Napier's treatment is based on the comparison of velocities of two moving points. Suppose one point P starts from A on line Ax of indefinite length with a uniform velocity V; then suppose another point Q starts from B on line By of given length r, at the same time that P sets out from A, and at the same velocity as that of P. Q, however, does not move uniformly, but its velocity changes in such a manner that at any point D it is proportional to the distance from D to y, the end of the line By. In this manner the velocities for equal intervals of time form a geometrical decreasing progression. If now C is

the point that P has reached, moving with uniform velocity V when Q, moving as described, has reached D, then the number which measures AC is the logarithm of the number which measures Dy.



The length Dy is called by Napier the Sine, and the whole line By is referred to as the Whole Sine. This term was not regarded in his time,nor for many years after, as a ratio, but as the half length of the chord of a circle of given radius which subtends the given angle at the center. Napier took the radius of the circle as 10^7 units, so that the sine of 90°, called by him the whole sine, is 10^7. The sines of the angles from 90° to 0° decrease in value and finally reach 0. His tables, therefore, represent the logarithms of numbers from 0 to 10^7. Since exponents were unknown, he did not use a base, the Naperian base e being entirely unknown to him. It might be noted that if Napier's system were referred to any base, that base would be more nearly equal to $1/e$. The exact relation has been computed and the following result obtained:

$$\log x = 10^7 \log_e 10^7 - 10^7 \log_e x,$$

in which log x represents the value of the logarithm of x as obtained by Napier.

To develop his canon from which the logarithms were computed, Napier first formed three tables of numbers, following the suggestion that the numbers shall form a geometrical progression. The first table consists of 101 numbers of which 10^7 is the first, each succeeding number being $1/10^7$ less than the preceeding, thus forming a geometrical progression with a common ratio of $1-1/10^7$. The numbers thus formed were arranged as follows:

10,000,000.0000000
9,999,999.0000000
9,999,998.0000001
9,999,997.0000003
9,999,996.0000006
. . . .
. . . .
9,999,900.0004950

The second table consists of a set of 51 numbers, beginning with 10^7 with a decimal of 6 zeros added. The table then "proceeds through

50 numbers decreasing proportionally in the proportion which is easiest, and as near as possible to that subsisting between the first and last numbers of the first table." Here it was found best to decrease the numbers each time by $1/10^5$, thus forming a geometrical progression with $1-1/10^5$ as the common ratio. Thus the second table is made up as follows:

10,000,000.000000
9,999,900.000000
9,999,800.001000
9,999,700.003000
. . . .
. . . .
9,995,001.224804

The third table consists of 69 columns, and in each column are placed 21 numbers, "proceeding in the proportion which is easiest, and as near as possible to that subsisting between the first and last numbers of the second table. Whence the first column is very easily obtained from the radius with four zeros added, by subtracting its 2,000th part, and so for other numbers as they arise." The first number in each of the columns is then determined by starting with the radius in the first column and "using the proportion easiest and nearest to that subsisting between the first and the last numbers of the first column." For this purpose it is found convenient to use as a common ratio 99/1000. Similarly, by using the same ratio a progression is made from the second number of the first column through the second numbers in all of the columns, this plan being continued throughout the 21 numbers of the first column. Napier then suggests that "it now remains in this third table to place beside these sines, or natural numbers, decreasing geometrically, their logarithms or artificial numbers increasing arithmetically." For this purpose Napier made use of two laws of limits which he developed. Also we find that he used the product, quotient and power laws, not in the form of formulas, but each was written out in full. The completed form of this third table was called by him the Radical Table. By use of it and the laws above mentioned he was able to construct the Principal Table with "great ease and no sensible error,"—a table which he called "The Cannon of Logarithms."

It is to be noted that, according to his system the logarithm of a number increases as the number decreases. For example the logarithm of 10^7 is zero, while the logarithm of a number about one half as large 5,038,768.7435, is 6,854,230.8. In view of the definition of the sine of an angle previously mentioned, it was a very easy matter to adapt

his cannon to the logarithms of the sines of angles. The Descriptio, in which Napier sets forth the wonderful invention, states and explains the rules, and applies them to trigonometric calculation. The final tables give sines and tangents of all angles from 0° to 90°.

It is remarkable that the worth of such an important invention was recognized at once. Henry Briggs, Professor of Mathematics in Gresham College, London, under date of March 10, 1615, writes: "Napper, Lord of Merchiston, hath set my head and hands a work with his new and admirable logarithms. I hope to see him this summer if it pleases God, for I never saw a book which pleases me better nor made me more wonder." It is also recorded that Briggs read it again and again, and oft explained it to his students. Briggs was very anxious to see Napier and discuss the system with him. With this end in view he journeyed from London to Edinburgh. History states that "these two men just stood and looked at each other for at least 15 minutes" when Briggs first entered Napier's study. On the continent Kepler's enthusiasm was almost equal to that of Brigg's.

In Brigg's first letter to Napier he suggested a division of the line into tenths, thus introducing the idea of the base 10. In his response to this letter, Napier suggested that it would be convenient for the logarithm of unity to be zero. With these two suggestions in mind, Briggs set about the calculation of a new system of logarithms, essentially the same as we now use.

Because of an admonition printed on the last page of some of the copies of his work in which Napier refers to the changes in the system, but does not give Briggs any credit, some historians have drawn the conclusion that Napier wished to retain all credit. That this is manifestly false lies in the fact that he always referred to Briggs as his "best beloved friend," and to his last days Briggs spoke of Napier in terms of warmest affection.

The strain involved in the computation and perfecting of the Cannon had been too great, and Napier did not long survive its completion, his death occurring on the 4th of April, 1617, just three years after the publication of the Constructio in which he described the nature of logarithms. He was one great inventor who lived to know that his works were appreciated.

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A PEAK INDIVIDUAL DONATION!

Norman E. Rutt, Professor of Mathematics, Louisiana State University, Baton Rouge, Louisiana, on October 21, 1943 donated $50.00 to the cause of NATIONAL MATHEMATICS MAGAZINE.

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Next Steps in Education and in the Teaching of Mathematics*

By WILLIAM BETZ
Rochester, N. Y.

LOOKING AT THE PRESENT SITUATION

Taking Inventory. When the war came upon us with the suddenness of a cyclone, the long crusade against mathematics had practically achieved its main objectives. The majority of our states had eliminated mathematics as a required high school subject. The peculiar doctrine had been given universal currency that, except in the case of a rather small group of technical or pre-engineering students, mathematical training had better be deferred. For all others, so it was loudly proclaimed by influential educators, administrators, curriculum "experts" and school counselors, the customary courses in "academic" mathematics were either too "hard", or not sufficiently "functional." Above all, they were not "socialized," and failed to meet the "immediate needs" of the adolescent population. It was declared with confidence that mathematical skills which the average worker might find indispensable in real life could readily be "picked up on the job, when and if needed." All protests, especially those based on cultural considerations, were either rudely dismissed as an impertinent interference by "vested interests," or were roundly ridiculed as the last futile gasps of the hoary myth of mental discipline, long since "exploded." As a result of all these attacks, mathematical enrollment figures during the decade preceding the war were going down with ever increasing speed.[1] Algebra and geometry were on the road to

*Based on an address delivered at the weekly Mathematics Conference of the summer session at Teachers College, Columbia University, on August 5, 1943.

[1] Thus, in New York State, the number of students registered in algebra *declined* from about 96,000 in 1928 to 65,000 in 1934, while in geometry the corresponding figures were 64,000 and 43,000 respectively. During this period the total school population in the same state increased as much as 46%. The shrinkage in mathematical enrollment figures continued unabated everywhere, until the arrival of the national crisis.

becoming college subjects. It was estimated that a year ago at least one and a half million students in our high schools were totally devoid of mathematical training beyond a very shaky acquaintance with elementary arithmetic.

Effect of the National Crisis. Very naturally, the national crisis found us unprepared. Almost overnight we faced the immense job of providing adequate mathematical and scientific training, not only for millions of soldiers, marines, and pilots, but also for that great host of workers who must stand back of them in our arsenals and defense industries. This improvised and belated training has been going on, at enormous cost, in army camps, pre-flight schools, summer schools, in evening courses, correspondence courses, and in many secondary schools. We are doing the best we can to correct the appalling mistakes of the past. With dramatic abruptness the great assault on mathematics has been stopped by the overwhelming actuality of a world in chaos. The bitter irony of a new epoch has swept aside, with relentless realism, the oratorical and uninformed declarations of educational theorists. By this time even the man on the street knows that mathematics is the sharpest weapon of mechanized warfare. Together with science, it enters the war effort at practically every point.

Facing the Future. It is not too early to raise the crucial question whether these impressive developments will at last mark an actual turning point in our official attitude toward the weighty problem of mathematical literacy and preparedness. Even now, there is some evidence that our dominant educational pressure groups, perhaps merely in temporary and unavoidable retreat "for the duration," intend to resume their customary policies as soon as we have returned to a peace basis. In that case, shall we again allow dubious slogans to jeopardize the security of the nation and the educational future of our young people? Or shall we have learned, once for all, that a citizen of the modern world cannot afford to be ignorant of mathematics? Obviously, these issues are both vital and extremely comprehensive. In the last analysis, they really pertain to elements of truth or error in our entire educational philosophy. Hence the restoration of mathematics to its rightful place in our schools is not a surface problem which can be settled by a few minor curricular adjustments. On the contrary, if a real cure is to be effected, there will have to be a fundamental re-examination of our basic objectives, as well as a corresponding rebuilding of the entire educational structure from the bottom up.

Need for a Coöperative Approach. No single individual, or group of individuals, may expect to deal adequately with so involved a

situation. Most certainly, only a coöperative and concerted effort is likely to produce the creative insight and the practical wisdom which we need in our struggle with the nationwide task of initiating a sound educational program for the critical years that lie ahead, and especially for the post-war period. And so, the pages that follow are certainly not be regarded as another "blueprint for the future." They were written primarily in the hope that many others might be induced to examine our pressing problems with care and to offer pertinent suggestions as to the "next steps."

It is the thesis of this paper that we are facing a dual job. *First,* we must find ways and means of reducing the present chaos in education. *Second,* we must build a mathematical program that will command greater respect, and we must bring our outmoded classroom procedures into harmony with enlightened pedagogic practices. The successive sections of this paper are devoted to a discussion of these two propositions.

PART ONE. NEXT STEPS IN EDUCATION

Two Contrasting Views. In discussions of the educational scene, it has been standard practice to dwell heavily on the amazing growth of enrollment figures, on the vast expansion and improvement of the school plant, on the ever increasing adaptation of the educational machinery to the needs of our diversified population, and the like. All these things are certainly indicative of great progress, and justify feelings of sincere satisfaction and gratitude. But when we turn from this splendid external picture to the inner mechanism of the school, to its offerings, its classroom procedures and its tangible results, the total impression is far less reassuring. Careful observers have traced most of the troubles of our schools to three main defects, as follows: (1) serious flaws in our educational doctrines and practices; (2) faulty organization of the educational system; and (3) inadequate preparation of teachers. In their opinion, the correction of these weaknesses will require the transforming measures mentioned below.

1. The Clarification of our Educational Policies

Looking at the Foundations. The Forty-first Yearbook of the *National Society for the Study of Education* (1942) presents a review of five current philosophies of education. The list of such "philosophies" might, of course, be extended. Terms like ***naturalism, experimentalism, realism, pragmatic instrumentalism, humanism, idealism,*** appear at every turn in educational discussions. To the initiated, each suggests a characterisitic orientation concerning the ultimate bases of reality, concerning the problem of values, as well as the nature and the desirable

outcomes of human evolution. That these philosophies are very far apart in many of their tenets becomes apparent upon closer scrutiny. Thus we have the naturalism and goalless experimentalism of the Dewey group at one end of the line, and the idealism of Horne at the other end, with the metaphysical authoritarianism of a Hutchins and the realism of a Breed somewhere between these extremes. To ignore or minimize the profound differences of these positions would be absurd. The attempt to fuse them or to effect a compromise of such opposites is bound to be futile. And yet the schools have been led to a sort of indiscriminate and blind eclecticism, even to the extent of a simultaneous and hence ludicrous espousal of mutually exclusive doctrines. For example, lip service is rendered to Deweyism one minute, but autocratic prescriptions are issued the very next minute. "Progressive" and "conservative" policies alternate in profuse array. "Individual needs and interests" nominally hold the center of the stage, but uniform tests follow at the close of the school year. The cult of the individual, of "personality", of free and uninhibited development is constantly accented in syllabi and in administrative bulletins, but is promptly disavowed by the formation of large and unassorted classes which reduce the individual to the level of the statistical averages of mass production. There must be no "failures," and yet we talk about standards of achievement. The common practice of a wholesale or automatic promotion of pupils robs the teacher of a foundation for that *continuous* growth in which we profess to believe. Worst of all, we are left without accepted guiding principles in the field of ethical conduct. The "scientific method" has done phenomenal service in the creation of our "new civilization," but to glorify it as a substitute for age-old ideals and the "eternal verities" leaves a spiritual vacuum that has not been filled. It is overlooked that a consistent materialist must deny the freedom of the will as well as moral responsibility. And there can be no doubt, as Bertrand Russell frankly admits in his own case, that a radical empiricism can only lead to "the debris of a universe in ruins" and hence to the dubious "foundation of unyielding despair."

The net result of all this is a confusion that borders on chaos. Like Mark Twain's famous horseman, school people are vainly trying to gallop bravely in all directions at once. But one cannot be a pragmatist on Monday, an idealist on Tuesday, and a realist on Wednesday, and yet steer a tolerably straight course. Dewey himself has repeatedly reacted to this deplorable state of affairs, and in no uncertain language. Thus, in his well-known Inglis lecture, delivered more than ten years ago, we find statements such as the following: "It is unnecessary to say that we are in the midst of great educational un-

certainty, one probably unparalleled at any past time. There is nothing accepted as axiomatic, nothing beyond the possibility of questioning, and few things that are not actually attacked. ... Anyone can bring forward a definition of education, but there are few who would not admit that the definitions when brought face to face with the actual conditions are likely to be hollow and empty."[2] He goes on to say: "As far as the uncertainty about standards, purposes, tendencies, and methods leads to discussion, there is something healthy about it.... But mere confusion is not a good thing.... Confusion tends toward obscuration of mind and ideas that leads to futility in action."

Since the issuance of these and similar pronouncements, an ever-increasing stream of critical appraisals of our educational philosophies has been forthcoming. The idolatry once accorded to popular educational slogans and magic formulas is waning. Instead, a more sober type of reflection is in the making.[3] It is being realized that opposites cannot be harmonized. And thus we are beginning to take the first and most crucial step in the direction of educational sanity, that of getting our bearings and deciding where we wish to go.

The Battle of Objectives. The fact that "at the present time education has no great directive aim" has led to uncertainty all along the line. If our confusion, according to Dewey, "is due ultimately to aimlessness," we really cannot expect a clear formulation of "objectives" until we have achieved a new vision of what we are trying to do. And without a body of basic objectives, "we don't know which way to go."

The present situation is described by Professor Edgar W. Knight of the University of North Carolina as follows: "Confusion of aims is today a very striking characterisitic of education in the United States. The numerous aims of education proposed and officially formulated and published have led to the confusion of teachers, administrators, and parents. More than fifteen hundred social aims of the study of English, more than three hundred aims of arithmetic in the first six grades, and more than eight hundred generalized aims of the social studies have been listed here and there in courses of studies and special

[2] Dewey, John, *The Way Out of Educational Confusion*, Harvard University Press, 1931, p. 1.

[3] Thus, critical reviews of our prevailing theories of education have begun to appear. See, for example, Schilpp, Paul A., editor, *The Philosophy of John Dewey*, Northwestern University, 1939; Kandel, I. L., *Conflicting Theories of Education*, The Macmillan Company, 1938; Breed, Frederick S., *Education and the New Realism*, The Macmillan Company, 1939; Justman, Joseph, *Theories of Secondary Education in the United States*, Bureau of Publications, Teachers College, Columbia University, 1940; Berkson, I. B., *Preface to an Educational Philosophy*, Columbia University Press, 1940.

studies."[4] Contemplating this picture, a witty critic has said that school teachers have many aims but no hits. A further step in the direction of improvement must therefore be the reduction of our bewildering assortment of objectives in favor of a few convincing, simple and common-sense agreements as to our line of march.

The Curriculum Muddle. It takes more than ordinary courage, patience, and experience to find one's way through the labyrinthian maze of our ever-growing curriculum literature. Entire libraries have accumulated around the "reconstruction of the curriculum." Every imaginable vagary, cult, or fad tries to find access to the long-suffering curriculum. Begining in a small way some thirty years ago, the passion for revising curricula was gradually fanned into a hot flame by enterprising institutions and curriculum "experts." At a safe distance from the classroom, they began to develop and to publicize an unending series of curriculum "approaches." Thus it was that "building" new curricula became a major industry. Eventually every town and hamlet felt the moral urge to construct its own streamlined courses of study in accordance with the latest prescriptions. The resulting phenomenal output of documents is being assembled in special centers called "curriculum laboratories," where they are being "evaluated." Thus, Dr. H. B. Bruner and his staff at Teachers College have now amassed a collection of nearly 85,000 courses of study, of which nearly 9,000 are devoted to mathematics.[5] To cope with this enormous "literature," they have developed more than one hundred "criteria of evaluation," as well as an elaborate system of indexing their findings. Significantly enough, of 28,477 special subject courses of study that have been examined, only 22.4% were pronounced "outstanding." As might have been expected, the curriculum revision movement reflects at every point the confusion mentioned above. It is here that experimentalism and "traditionalism" are in sharpest contrast. The exponents of "change" are pitted in solid array against the defenders of tested permanent values. The advocates of "emerging" curricula based on "experience" turn with scorn from organized and sequential knowledge of the "set-out-to-be-learned" variety.[6] Extremists have not

[4] Knight, Edgar W., *Progress and Educational Perspective*, The Macmillan Company, 1942, p. 126.

[5] See Bruner, Herbert B. and others, *What Our Schools are Teaching*, Bureau of Publications, Teachers College, Columbia University, 1941, p. 9.

[6] A most revealing indication of what is meant by an "experience" curriculum, or an "emerging core curriculum," may be found in publications such as the following:

(1) Mississippi Program for the Improvement of Instruction; *Curriculum Reorganization in the Secondary School, Grades 7-12.* State Department of of Instruction, Jackson, Mississippi, 1939.

(2) *Living, the Basis for Learning*, from the Santa Barbara City Schools Curriculum Laboratory. Published by Educational Factors, Santa Barbara, California, 1942. (Profusely illustrated.)

hesitated to picture the "planless" school as the ultimate triumph of curriculum revision: quite true, for the curriculum will then have ceased to exist and will no longer trouble us.

Grave concern has been expressed by trustworthy educational critics over the impending fragmentation, not to say dissolution, of our national culture, if we continue in this direction. The national crisis has served to highlight the danger of forgetting to include in the cultural equipment of our children and young people a dependable knowledge and understanding of the fundamentals, of our common heritage, our history and ideals, and the American way of life. And in the coming "air age", education must effect an even wider perspective which should embrace global backgrounds, adequate economic and scientific orientation, and an insight into the complex machinery of our industrial and technological civilization.

Now, two essentially different attempts have been made in recent years to secure a greater degree of unification for our disintegrating curricula. The elementary school has learned to stress "centers of interest," while the secondary schools are experimenting with "core curricula" or various "integrated" programs. But only too often the "centers" have no connecting bond and the "core" has no central theme. Sensing these defects, and being keenly aware of the approaching world crisis, a militant group of "frontier thinkers" lost no time in advancing their revolutionary ideas. They insisted that the schools become actively concerned with *social* reconstruction and that they assist in "planning and building the new society." Henceforth, "social problems" were to be the primary interest of the school.[7] "Progressive" educators, at first attracted by this arresting proposal, eventually turned away from it. A "planned society," they felt, would cramp the style of the individual, and that just could not be tolerated.

But there is another, less precarious road to a unified curriculum. Thus, along with thinkers like Whitehead, one may follow the far more realistic plan of basing the integration of the curriculum on the inclusive concept of "Life in all its manifestations." In that case one has not only a unitary theme of broadest possible scope, but also a set of constituent and closely related factors, each of which must be given its due share of attention. Instead of an arbitrary and ever changing collection of odds and ends, there now arises a body of permanent categories such as common human needs, the study of the

[7] See, for example, Kilpatrick, William H., editor, *The Educational Frontier*, D. Appleton-Century Company, 1933; Counts, George S., *The Prospects of American Democracy*, The John Day Company, 1938; Everett, Samuel, editor, *A Challenge to Secondary Education*, D. Appleton-Century Company, 1935; Rugg, Harold, *American Life and the School Curriculum*, Ginn and Company, 1936.

environment, vocational competence, training for citizenship, and all-around cultural and intellectual growth. One of the best-informed recent studies of educational trends, that of Berkson, offers the following appealing summary of acceptable functions of a desirable curriculum.[8]

"1. Transmission of the social heritage of knowledge, skills, and attitudes
2. Maintenance of good health and balanced development of physique.
3. Training of the individual character.
4. Preparation for making a living.
5. Cultivation of the personality, through art, music and literature.
6. Development of the desire and the ability to think.
7. Understanding of social problems, development of social sensitivity, and inculcation of a sense of social obligation."

Theories of Learning. Hardly less disturbing than the chaos of the curriculum is the prevailing confusion in the field of *methods* of teaching. The fact that very significant changes have occurred in our conception of the learning process during the past two decades is still news to a very large number of teachers. Ranging all the way from a purely mechanisitic assignment of lessons, with the demand for a slavish reproduction of the textbook, to an unguided and hence wasteful roaming over uncharted "life situations," our classroom procedures, only too often, cannot possibly be characterized as either efficient, or economical, or inspiring.

Few will deny that it does make a tremendous difference how a subject is approached, presented, and developed in the classroom. Hence it is most heartening to note that recent developments in educational psychology are in harmony with the dictates of common sense and with the recommendations of national committees.[9] The coming reorganization of education will certainly have to give more than casual attention to an enlightened methodology of learning. Among the points that teachers will hereafter find it desirable to keep in mind are the following:

1. *The concept of learning* has been modified, enlarged, and made less mechanical. Learning is no longer defined merely as "connecting." (Thorndike).
2. The "bond theory" of learning, as an all-inclusive formula, especially in a mechanistic setting, is passé. With it goes the atomistic dissection of school subjects into "elements" to be mastered by *repetition*.
3. A new theory of *practice* or *drill* is being developed, which includes considerations of purpose, understanding, meaning.

[8] Berkson, I. B., *Education Faces the Future*, Harper and Brothers, 1943.

[9] See, especially, the Forty-first Yearbook of the National Society for the Study of Education, 1942, Part I, *The Psychology of Learning;* Gates and others, *Educational Psychology*, The Macmillan Company, 1942; Katona, G., *Organizing and Memorizing*, Columbia University Press, 1940; and *Arithmetic in General Education*, Sixteenth Yearbook of the National Council of Teachers of Mathematics, 1941, Chapters IX-XI.

4. The concept of *maturation* has caused a new appraisal of grade-placement problems.

5. "Specific" learning, at least in mathematics, is conceded to be far less effective than conscious *generalization*.

6. Intrinsic *motivation* is seen to be an absolutely essential condition of real learning.

7. Learning involves *organization*. It includes both analysis and synthesis, differentiation and integration.

8. *Meanings*, concepts, principles are the real carriers of "transfer," *not specific skills* or facts.

9. Not only is actual "*transfer*" *of training* possible; it represents the *central challenge* of the teacher.

10. Efficiency in *problem-solving* is now stressed more than ever as a primary essential of productive classroom techniques.

11. *Social situations* or life problems are to be regarded as vital both for purposes of immediate motivation and of extensive "transfer."

12. The new "field" theories of learning (configuration, Gestalt) have served to make clear the vast importance of *backgrounds* and *total situations*. Without organic over-views and integrating principles, there can be no perspective and hence no real comprehension. Isolated details are not the "common carriers" of a year's program or of an entire course of study. Teachers should be directors of a symphony and not mere drill sergeants of unrelated odds and ends.

2. *Reorganizing the Secondary School*

The Outmoded Four-Year High School. It is an undeniable fact that for many years the traditional four-year high school virtually made impossible vitally necessary reforms in the organization of the school system. A high school diploma resting on the outworn concept of precisely sixteen "units," many of them begun too late, ruled out either genuine cultural growth or adequate vocational training. When mass education appeared on the scene, the situation became much worse. The secondary school was declared to be the "people's university," offering "everything to everybody." The elective system followed, together with a disregard for standards. But unless the school wished to invite complete chaos, it soon found itself up against the old basic questions of aim, content, and organization. Above all, the vastly expanded interests of the "new" high school simply would not fit into the rigid framework of a four-year sequence.

The Reorganization Movement. The story of the gradual transformation of our school system is, perhaps, too well known to require rehearsal at this point. Here again, however, we have another typical case of confusion. In addition to the traditional 8–4 plan, we now have 6–3–3, 6–6, 7–5, 6–4–4, and other plans. In a recent year, according to a report of the United States Office of Education, the 25,000 public high schools exhibited 29 types of organization, five of these being of the "regular" type, and 24 of the reorganized pat-

tern,—that is, schools which have deviated from the more conventional pattern of having 7 or 8 years in elementary school followed by a single-unit four-year high school. During that year, nearly three-fifths (61.3 per cent) of the high schools reporting were of the "regular" type. The junior-senior pattern was followed in 13.1 per cent of the schools and the undivided in 12.1 per cent; 9.6 per cent were junior high schools; and 3.9 per cent were senior high schools. But the percentages of pupils enrolled in various types of high schools, as given by the following table, tell an even more striking story:

	Junior	*Junior-Senior*	*Senior*	*Regular*
1930	19.0	17.9	9.9	53.2
1934	18.6	18.9	11.3	51.2
1938	19.0	24.4	13.1	43.5

The significant fact is that the enrollment in reorganized schools now exceeds the enrollment in "regular" schools.

Need of Further Adjustment. While these developments are certainly in the right direction, they have not gone far enough. The gradual elimination of the traditional high school must be supplemented by much more careful provision for our most gifted children. It is true that we have committed ourselves to a unitary system of education, as compared with the European dual system. But we cannot ignore the enormous waste resulting from neglecting many thousands of superior children—the potential leaders of the nation—who could easily complete their secondary and professional training at a much earlier date while yet obtaining a more thorough and significant type of education. Throughout the years, there have been vigorous champions of their cause, ever since the days of Thomas Jefferson.[10] To date, little has been done to improve the situation. The fearful handicap of a postponed education, from a standpoint of health, finance, and ultimate efficiency, has not yet dawned upon those who seem to be interested exclusively in the lower intelligence levels. One of the most urgent "next steps" in education will have to be the correction of this tragic oversight. In the larger communities the best solution would seem to be that of providing at least one high school of the reorganized type, open only to those found to be qualified, and based entirely on merit and real achievement.

[10] See, especially, the Twenty-third Yearbook, 1924, Part I, of the National Society for the Study of Education, on *The Education of Gifted Children.*

3. *Providing a New Outlook for Teaching*

A Story of Ups and Downs. Back of our present teacher training programs is a record of both phenomenal progress and of deplorable stagnation. Step by step the opportunities for professional growth have been extended. Today the total resources placed at the disposal of teachers are fairylike in comparison with the utter barrenness of former years. It is an undeniable fact, however, that there is still much evidence of unpardonable backwardness.[11] This unfortunate condition is undoubtedly due in large measure to such factors as the initial preparedness of teachers, the mode of selecting them, the absence of adequate supervision and of further in-service training, as well as an insufficient financial reward. Taking the prescribed number of college courses, and being credited with the requisite number of "points" and "hours", is no guarantee either of an adequate cultural background or of fitness for teaching. But even if it is admitted that teachers are born, not made, there is no substitute for thorough and long-continued study and professional training on the part of prospective teachers. While there is still considerable variation in our teacher-training procedures, authoritative committees have worked out dependable recommendations which will tend to raise the standards of the teaching profession and thus to correct an unsatisfactory situation.[12]

Toward New Horizons. Among the "next steps" that will greatly improve the whole outlook of the teaching profession are the following:

1. Courses in "*methods*" *of teaching* should be entrusted only to teachers who have had successful experience in the classroom.
2. There should be a larger number of genuine *laboratory schools* in which sound classroom procedures are developed, demonstrated, and evaluated by superior teachers.
3. A sufficient amount of successful *practice teaching*, under competent supervision, should be required of all prospective teachers.
4. Regular *in-service training*, at minimum expense to the teacher, should be made available at least in all large school systems.

[11] See, for example, Mort, Paul R., and Cornell, Francis G., *American Schools in Transition*, Bureau of Publications, Teachers College, New York, 1941, Chatper XI.

[12] Thus, the publications of the Commission on Teachers Education of the American Council on Education (744 Jackson Place, N. W., Washington, D. C.) throw much light on that area. See, for example, *A Functional Program of Teacher Education*, Washington, D. C., 1941.

In the field of mathematics, the Report of the National Committee on Mathematical Requirements (1923) contained an exhaustive study on *The Training of Teachers of Mathematics*, based on international backgrounds, by Professor R. C. Archibald; the recent Report of the Joint Commission (*op. cit.*, 1940) devoted a special chapter to *The Education of Teachers;* and a commission of the Mathematical Association of America published a *Report on the Training of Teachers of Mathematics* (American Mathematical Monthly, 1935, pp. 263-277).

5. *Salary schedules* should be more nearly commensurate with increased professional requirements, and with the actual cost of living.

6. *Security of "tenure"*, after a probationary period, should be guaranteed by state laws, as well as provision for reasonsble retirement annuities.

On such a basis, properly qualified teachers may be expected to meet the challenge of the new age. It remains true that in teaching, as well as in every other field, we reap no more than we sow: "As is the teacher, so is the school."

From the picture outlined above it should have become apparent that, all along the line, there is much room for improvement. Our educational policies should certainly be characterized by less vagueness; we should have more common sense and less nonsense; we should not allow personal whims or preferences to rule out tested backgrounds and race experience; and we should substitute order for aimlessness. Classroom procedures should be based more definitely on rational theories of learning. Greater attention should be given to the elaboration and the enforcement of reasonable standards. Better provision should be made for economy of time and continuity of growth, especially in the case of gifted children, through a reorganization of the school system. And our constant endeavor should be in the direction of awakening a deep and genuine devotion to lofty ideals of conduct and of service.

The greatest pragmatic test of all history, that of the present war, has been an amazing revealer. It has amply vindicated those who have stood for substantial, consecutive, and thorough training, and who have stressed high standards of achievement. Every day of the titanic struggle has proved that brave men and women are fighting and suffering, not for a scientific "hypothesis," or for a personal "experiment" as to the nature of patriotism, but for loyalties too deep for words.

PART TWO

NEXT STEPS IN THE TEACHING OF MATHEMATICS

The Long Struggle for Improvement. A general acquaintance with recent developments in the teaching of elementary and secondary mathematics is presupposed in the following pages.[13] We have already referred to the pre-war crusade against mathematics, its causes and its serious effects. If the national crisis should continue for a considerable period, it will bring about a strengthening of the funda-

[13] See, especially, the *Tenth, Eleventh, Fifteenth,* and *Sixteenth Yearbooks* of the National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University; also, numerous articles in *The Mathematics Teacher*.

mentals all along the line, as well as far greater emphasis on military, aeronautical, and industrial applications. Should peace be restored more promptly, however, the critics of mathematics will undoubtedly return to their old slogans,—"immediate needs and interests," "social problems," "functional values," "minimum essentials," and the like.

On the other hand, if the lessons learned in this war emergency should really force us to carry out such broad corrective measures as were suggested above, then it will be possible to build for mathematics the new foundation that has been the vision of mathematical leaders for five decades. What an impressive series of proposals, studies, and reports has been prepared during that period! Beginning with the Report of the Committee of Ten, way back in 1893, and the early reform movement inaugurated by Professor E. H. Moore's famous address of 1902, efforts at improvement have been continuous.[14] The 1923 Report of the National Committee on Mathematical Requirements, sponsored by the Mathematical Association of America, was a landmark. Again, since its organization in 1920, the National Council of Teachers of Mathematics has been a consistent advocate of mathematical reform in the secondary schools, through its yearbooks and through a large number of meetings, addresses and published papers. Recently, the two Yearbooks on Arithmetic, and the Report of the Joint Commission on the Place of Mathematics in Secondary Education (Fifteenth Yearbook, 1940) have tried to clarify and correct conflicting trends, and to offer dependable plans for the future.

In résumé, it must be said that while there had been much real or potential progress during this period, the pre-war epoch closed with sharp disharmonies. Arithmetic had become the football of educational extremists, junior high school mathematics had become devitalized, and all "academic" courses beyond the eighth grade were either being "stepped up," or "socialized," or eliminated. But for the war, that is where we should find ourselves today.

Let it be repeated, therefore, that the necessary reconstruction of mathematics is a coöperative job which demands the honest assistance of educators and administrators. Unless their negative attitude is reversed by *public protests* and by *imperative demands from the American people*, we shall continue to have merely surface changes.

1. *Major Steps Toward a New Mathematical Program*

Seven Necessary Steps. A review of the most authoritative discussions having to do with the reorganization of mathematics clearly points to the following central issues:

[14] For an able review of these efforts during the first 25 years of our century, see Professor D. E. Smith's paper, *A Survey of Progress*, in the First Yearbook of the National Council (1926).

1. We must have an ***organically planned and integrated curriculum*** which extends continuously from the Kindergarten to the upper levels of the high school.

2. Arithmetic must be taught from the beginning as a ***system of ideas***, to be mastered by insight and meaningful applications.

3. ***Informal geometry*** must become an ***integral part***, from the earliest grades, of a cumulative trunk-line curriculum.

4. ***Algebra*** and ***formal geometry*** must no longer be restricted to one-year courses, but must be distributed over a longer period in the interest of real understanding and mastery.

5. Indispensable aspects of ***modern mathematics***, which have long been given adequate recognition by other leading countries, should be made available at least to gifted students.

6. The work of the classroom must stress continuously the ***place of mathematics in modern life***, by systematic emphasis on significant applications within the learner's comprehension.

7. The ***teachers of mathematics*** must themselves attain a greater acquaintance with pure and applied mathematics, and must free their subject from the tyranny of organized drudgery and blind manipulation, in favor of real understanding and purposeful application.

2. *Comments on the Redirection of Mathematics*

A Long-Range Period of Reconstruction. It is clear that such a program, however urgently it is needed, will require much reorientation. It will take time before the opponents of mathematics will really grasp the new situation. It may take even more time to obtain concerted action. But without a definite plan little or nothing can be accomplished. Hence we turn to some explanatory comments, on various aspects of the present situation, that may help to expedite the necessary redirection of mathematics.

The Redirection of Arithmetic. The war has suddenly turned a pitiless spotlight on the shortcomings in that instructional area. Almost incredible, wholesale imcompetence in the simplest computations has been revealed by the screening tests given by the armed forces. To teachers of mathematics these revelations occasioned no surprise. They merely served to publicize what the teachers had proclaimed on innumerable occasions, without noticeable effect. "For the duration," at least, the order has gone out from the military authorities that these weaknesses *must* be corrected, and *at once.* It seemed to come as a shock to educators that a subject which they had treated in such a cavalier way should be so vitally necessary for purposes of national defense. All the usual slogans suddenly evaporated. And so, schools and colleges arranged feverishly for arithmetic "refresher" courses. This designation has been received with much sarcasm by teachers, who have pointed out that you cannot "refresh" what never existed. Several authoritative reports have stated bluntly

that the nationwide disabilities in arithmetic are obviously not due to "forgetting," but to lack of understanding and of basic training.[15]

What then, should be our line of march in the field of arithmetic? First of all, it seems obvious that teachers and administrators should acquaint themselves with the causes that are back of the collapse of arithmetic, which have long been known.[16] They should then abandon the false theories of educational extremists. In particular, they should no longer regard arithmetic as an incidental and more or less negligible by-product of "activities" and "social" situations. Finally, careful attention should henceforth be given to the competent advice of real students of arithmetic. The following statement might well serve as a good starting point: "Arithmetic is a system of ideas. It is not a collection of objects. It is not a set of signs. It is not a series of physical activities. ... It has nothing to do with the amount of chalk dust forty pupils can raise in a schoolroom in thirty minutes. Arithmetic exists and grows only in the mind. ... It is not learned according as the interests or the whims of pupils may suggest. It is not anyone's personal discovery or invention. *Arithmetic must be taught as a system.*"[17]

The Case of Informal Geometry. For generations the mathematical program of the elementary school has been limited almost entirely to arithmetic. Only the higher grades constituted an exception to the extent that it was customary to include some work in practical mensuration. This arrangement overlooked the fact that *mathematics has a dual foundation,—arithmetic and geometry.* Number and form, counting and measuring, appeared on the scene together, for number was the indispensable tool of measurement, and considerations of shape, size and position accompanied even the primitive artisan in all his practical activities. Thus it was that the geometry of everyday life, often called intuitive or informal geometry, preceded the development of demonstrative geometry by thousands of years. And it is this kind of everyday geometry which we must have as one of the cornerstones of the mathematical edifice. Its virtual omission has given us the lopsided, distorted curriculum to which may be traced many of our mathematical troubles in the high school.

[15] See, for example, the October, 1943 issue of *The Mathematics Teacher*, p. 244; also, the monograph entitled *An Analysis of the Arithmetic Knowledge of High School Pupils*, by Jacob S. Orleans and Emanuel Saxe, School of Education, College of the City of New York, 1943.

[16] Reliable information concerning this matter can readily be obtained from such sources as the *Tenth Yearbook* of the National Council of Teachers of Mathematics, on *The Teaching of Arithmetic*. See, especially, Chapter I, by Dr. William A. Brownell.

[17] From *Arithmetic in General Education*, the Sixteenth Yearbook of the National Council of Teachers of Mathematics, p. 80.

The arrival of the junior high school movement marked the beginning of a more serious effort to restore the "missing link" of mathematics to its rightful place, especially since the appearance of the National Report of 1923. Slowly, and often in very imperfect fashion, the necessary geometric material began to make itself felt in the textbooks and syllabi. But a great deal of improvement is still necessary.[18] The war situation may well mark a turning-point in our notorious indifference to the claims of geometric instruction. All pre-induction syllabi in mathematics have stressed the essential character of a dependable geometric training. They demand a knowledge of geometric forms and concepts, skill in direct and indirect measurement, familiarity with basic constructions, with scale drawing, blueprint reading, map reading, and the like. For nearly a century American mathematical leaders have demanded a coördination of geometry and arithmetic. That step has long been taken by all other leading nations. We cannot afford to postpone it any longer in our American schools.

What About Socialized Mathematics? So many different interpretations have been given of the concept of "socialized mathematics" that a universal explanation seems impossible. Fundamentally, however, it rests on the Dewey idea that "education *is* life, not preparation *for* life." In fact, the basic thought was expressed with clearness by none other than Professor A. N. Whitehead, when he wrote: "There is only *one* subject matter for education, and that is *Life* in all its manifestations. Instead of this single unity, we offer children—Algebra, from which nothing follows; Geometry, from which nothing follows; History, from which nothing follows; a couple of Languages, never mastered. . . . Can such a list be said to represent Life as it is known in the midst of the living of it?" Written *thirty years* ago, this statement was recently chosen by the Educational Policies Commission as a motto for one of its best known volumes.[19]

For a long time the feeling has been gaining ground that secondary education had become, as Whitehead expressed it, a domain of "inert ideas." The same thinker asserted that "education should be useful, whatever your aim in life," and he defined education as "the acquisition of the art of the utilization of knowledge." He safeguarded his position, however, by also making a plea for the development of Power, "an intimate sense for the *power* of ideas, for the *beauty* of ideas, and for the *structure* of ideas."

[18] For an extensive study of the development of informal geometry as a school subject see the writer's monograph on *The Teaching of Intuitive Geometry* in the Eighth Yearbook of the National Council of Teachers of Mathematics.

[19] See *The Purposes of Education in American Democracy*, 1938, Chapter III.

Instead of such a broad view we have had, for some years, an extremely narrow type of crass utilitarianism. "Social utility," "functional values," "immediate applicability," and similar phrases have become the transforming influence back of nearly all recent curriculum approaches. An appraisal of this movement, as applied to mathematics, was presented by the writer in two recent papers.[20] The great significance of life-situation problems was frankly recognized. But it was also stated that social backgrounds as such, as Professor McConnell has said, "do not force meaningful learning." It was shown that many attempts have been made, over a long period, to base the mathematical curriculum entirely on real problem situations,—social, technical and commercial—and that all these attempts have regularly failed. The reason for this failure was located in the fact that *mathematics is a cumulative system of concepts, principles, and techniques, while "life situations" are of necessity more or less unrelated and hence unsystematic.* And so, it will *always* be impossible, *within the customary time limits*, to derive the mathematical curriculum entirely from life situations.

On the other hand, the demand of "socialization" has forced mathematics to "get down to earth," and to give up much of its purely "academic" flavor, including not a few more or less unessential details. Above all, it has begun to create a much needed basis for *motivation*, and for a much wider degree of genuine "*transfer*." Nevertheless, only in the new, continuous curricula of the future will it be possible to effect a better compromise between theory and practice, between insight and effort, and between inspiration and drudgery.

The Outlook for General Mathematics. In the course of time the mathematical "Tree of Knowledge" developed, in addition to arithmetic and everyday geometry, the branches which are known by the names of demonstrative geometry, trigonometry, algebra, analytical geometry, calculus, descriptive geometry, and so on. Besides, the knowledge of "pure" mathematics became interwoven in countless ways with the practical affairs of modern life—with science, industry, technology, engineering, economics, sociology, and the like.

The prevailing curriculum ignores not only the organic relationships of the various branches of mathematics, but also their fruitful application. Instead, we still insist on studying each mathematical subject in a compartment by itself and in a manner which is neither psychological nor practical. Moreover, we crowd each of these compartmentalized courses into a single semester or a single year. And

[20] See *The Present Situation in Secondary Mathematics*, in The Mathematics Teacher, December, 1940, pp. 351 ff., and *The Necessary Redirection of Mathematics*, in The Mathematics Teacher, April, 1942, pp. 151 ff.

any pupil whose mental digestion is not equal to the strain of this forced diet is called a "failure."

The rebellion against these erroneous procedures was openly inaugurated by Professor E. H. Moore in his famous address of 1902.[21] It was many years, however, before textbooks and curricula showed the effect of this movement. At the college level we now have numerous texts for freshmen which embody quite a variety of orientation or fusion programs. In the secondary field only the junior high schools have succeeded in working out a composite approach, largely in grades 7 and 8. The regular four-year high schools have done little to encourage or develop *sequential* courses in "general mathematics." Among the reasons for this stagnation are the inhibiting factors pointed out in the first part of the paper. So long as educators and administrators refuse to endorse a *continuous* program in mathematics, real improvement is almost impossible. It should be entirely self-evident that a genuine course in general mathematics must provide for appropriate growth in arithmetic, geometry, algebra and trigonometry, and must include adequate attention to applications. It should therefore extend over a period of several years. *A one-year course in general mathematics is a pedagogic absurdity.*

The Ninth Year as a Storm Center. We have spoken of the considerable diversity in the organization of our secondary schools. That fact has served to make the ninth school year into an all-around educational headache. For it is here that all the different curricular types have either their inception, or their continuation, or their terminal point. In the $6-3-3$ school systems, the ninth year is a terminal year; in the $8-4$ plan, it is an initial year; and in the $6-6$ or $6-4-4$ plans, it is a transition year. We therefore find in the ninth-year mathematics courses an almost complete lack of agreement as to desirable or essential objectives. Thus, there are one-year terminal courses with a large variety of ingredients. There are "socialized" courses which rise little above eighth-grade arithmetic. Business courses and other "vocationalized" programs are offered in many specialized high schools. Before the war, the trend was away from the "regular" algebra programs of former years.

It should be apparent that we have here a very unsatisfactory and unstable situation. This was shown very clearly in a recent thesis for the master's degree written by Faith F. Novinger of Washington, D. C.[22] She made an intensive study of twenty-three mathematical

[21] This address was reprinted in the *First Yearbook* of the National Council of Teachers of Mathematics, pp. 32-57.

[22] The original study was placed at the disposal of the writer. A brief abstract was published in *The Mathematical Teacher*, April, 1942.

textbooks intended for ninth-grade pupils, published between 1934 and 1940. No regular algebra texts were included. Her findings revealed a state of confusion bordering on chaos, so far as the offerings and objectives of these books were concerned. Thus, the amount of space given to arithmetic in these texts varies from 4 to 209 pages; in algebra it varies from 4 to 207 pages. Seven of the texts omit algebra altogether. In geometry there is a closer agreement, and the range tends to cluster around 100 pages. A similar study, made by a committee of Rochester teachers, involved an analysis of fourteen one-year terminal textbooks in ninth-year mathematics, all published since 1935. In arithmetic, the percentage of text pages varied from 0% to 94%; in algebra, the range extended from .5% to 72%; in geometry, from .5% to 37%; in trigonometry, from 0% to 15%; in correlated mathematics (mainly socialized applications), from 0% to 51%.

The war crisis has served to expose the utter untenableness of this state of affairs. It has stressed the fact, which should have been obvious long ago, that *all* our young people need fundamental training in the *same basic mathematical elements.* Wide publicity has now been given by war committees as to these basic elements, and there is no longer any excuse for the kind of tinkering with mathematical curricula which has been going on for years to the detriment of the nation.

Whither Algebra? As has been suggested repeatedly, algebra has long been the pet aversion of educational authorities. It has often aroused their ire to an almost pathological degree. Thus, some years ago a well-known general educator declared that algebra was "morphine" for young people. That this greatly disliked subject is the very backbone of science, engineering, industry, technology, and is indispensable in an ever-growing number of other fields, is constantly overlooked. The war emergency has called for the immediate restoration of algebra to its rightful place, thus overruling the violent opposition it has encountered in our secondary schools.

It would be very foolish, however, to go to the opposite extreme, that of ignoring the grievances which are regularly associated with the customary curriculum in elementary algebra. The indictment of algebra is based, in the main, on the following counts:

1. The subject is too *abstract* to be of interest and value to young learners.
2. Its *techniques*, to a large extent, cannot be fruitfully applied at the secondary level, and hence are "*non-functional.*"
3. The usual *problem content* of elementary algebra is of little or no immediate value, having no apparent "functional" or "social" objectives.

4. The subject ignores, or postpones, mathematical areas that are *much more* "functional," interesting, and socially significant, such as "socialized" arithmetic, business and shop problems, science problems, and practical work in mensuration.

5. Pupils of the lower intelligence levels cannot profit from the orthodox algebra course; it is *too difficult and covers too much ground.*

This arraignment would lose much of its sharpness if the teaching of algebra were distributed over several years, as it is in all other leading countries. In a one-year course it is impossible to avoid completely all the criticisms mentioned above.

Nevertheless, even within these limitations (for which algebra alone cannot really be blamed) a good deal could be done to mitigate still further the usual causes of friction. For it would be unjust to deny that a great transformation has already occurred in algebra textbooks and in current classroom procedures. Among the steps in the direction of still greater improvement are the following:

1. The basic concepts and principles should be made more meaningful.
2. The essential techniques should be introduced more gradually and should not stress unusual types of work (e. g., long polynomials, complicated fractions, involved cases of factoring, of radicals, equations, evaluation, and the like.)
3. The useful or "functional" areas of the subject should receive heavier emphasis.
4. The problem content should be made more realistic and significant.

The Case of Demonstrative Geometry. Like algebra, demonstrative geometry has for years been under a cloud of almost universal condemnation. A very extensive literature has arisen as to the faults and the merits of this subject.[23] Dating back to the days of ancient Greece, and having attained at an early date an amazingly finished form, it has clung to this inherited form and organization with heroic and yet regrettable tenacity. It refused to go along with the forward march of mathematics. Its chief devotees paid little or no attention to the fact that in Euclid's day there existed, aside from his own great work, hardly any mathematics beyond crude forms of computation and of mensuration. But for several centuries we have now had algebra, analytical geometry, and the calculus. The nineteenth century saw the arrival of non-Euclidean geometries. And in recent decades there has been an amazing development in foundation theory and in symbolic logic.

As Professor Keyser pointed out long ago, the great contribution of Euclid was not geometrical, but methodological.[24] It was the first

[23] Thus, a recent bibliography of mathematical education, that of Professor William L. Schaaf, gives 272 titles of articles on the teaching of geometry, which have appeared in the periodical literature since 1920, See, also, the *Fifth Yearbook* of the National Council of Teachers of Mathematics, on *The Teaching of Geometry.*

[24] Keyser, C. J., *Thinking About Thinking*, E. P. Dutton & Co., 1926, p. 26.

recorded major attempt at systematic, autonomous thinking. And it is that kind of thinking which primarily justifies the place of demonstrative geometry in the curriculum.

It has therefore been clear for many years that the teaching of demonstrative geometry must be readjusted. But how? Many have been the efforts to reconstruct the ancient edifice. They have ranged all the way from an inconsequential tinkering with surface changes to complete transformation or the virtual annihilation of the whole structure. Present trends in the direction of reorientation are along the following lines:

1. The abandonment of deductive procedures in favor of an intuitive acceptance of geometric facts and of their extensive application in practical problems.
2. The retention of a minimum list of selected propositions, which are to be proved with various degrees of rigor.
3. The attempt to "transfer" geometric "modes of thinking" to non-geometric situations.
4. The attempt to refine the logic of geometric demonstrations.
5. An increasing use of analytical methods.
6. An even greater emphasis on so-called "originals," or else their drastic curtailment.
7. The fusion of plane and solid geometry, or the admixture of three-dimensional units.

Each of these trends has its own history. What makes the present status of geometry so vulnerable and unstable is the fact that many teachers and textbook writers, in their endeavor to be "up to date," try vainly to do nearly all of these things simultaneously and in a single year. The first step toward recovery in this field is that of becoming clear about the function of deductive thinking in human affairs. If one affirms the all-embracing role of this mode of thinking, it is still necessary to take the further step of investigating to what extent geometry may serve as a vehicle for this kind of training, and how this training may be "transferred." Only a beginning has been made thus far in these basic and unavoidable considerations.

Beyond Geometry. The armed forces are now urging that four years of mathematics beyond the eighth grade be made available for capable students. Engineering or technical schools have long insisted on the equivalent of that amount of mathematical preparation. What should be the nature of this preparation? The standard practice still is to offer, beyond elementary algebra and plane geometry, unrelated and mutually independent units of trigonometry, solid geometry, and advanced algebra. We know only too well, however, that in a large number of cases the technical or science problems which the student is expected to face require the coöperative use of *groups* of mathematical concepts or skills. The arguments in favor of a closer correlation of

the various mathematical categories are indeed most convincing, if not unanswerable.

But aside from this demand for *integration* there is the urgent need of including in the upper years of the curriculum those neglected mathematical areas which have long been incorporated in the mathematical program of many other countries. We refer to the elements of analytical geometry, of the calculus, of statistics and finance, and of geometric drawing involving a knowledge of descriptive geometry. Our consistent slowness in becoming up-to-date in mathematics puts our most promising young men and women at a great disadvantage. It crowds their college years beyond endurance, often retards their life work by at least two years, and imposes economic burdens of a grievous character. Should it not become an established policy to open the door of opportunity to those on whom we must depend for leadership in science, technology, and industrial planning?

Effect of the War on the Teaching of Mathematics. The national crisis has had the beneficial effect of giving great prominence to basic training in mathematics. The armed forces are now urging continuous attention to mathematics, beyond the eighth grade, on the part of all capable students, and they insist on a minimum preparation, for every recruit, that presupposes at least one year of composite work in arithmetic, applied geometry, elementary algebra, and trigonometry.[25] Besides, they have made it very clear that what is wanted is ***mastery and understanding.*** As a result, emergency courses have become necessary, and all manner of rapidly constructed syllabi and textbooks are being made available.[26]

Now, at least two very real dangers resulting from these developments must be watched with great care. The first has to do with the excessive pressure and acceleration that generally attends all emergency courses. Thus, one official syllabus in mathematics assigns 7 hours to arithmetic, 17 hours to algebra, 17 hours to geometry, and 19 hours to spherical geometry and trigonometry. How much real understanding and mastery can be achieved under such a plan? The second danger is that of crowding into all mathematical curricula a veritable avalanche of applied problems having to do with the war, with industry, science, technology, and so on. It should be emphati-

[25] The most authoritative recent report pertaining to this matter is the one published in the October, 1943, issue of *The Mathematics Teacher*, under the title *Essential Mathematics for Minimum Army Needs.*

[26] For a critique of the confusion which has been caused by conflicting opinions and hastily compiled war courses see, *Mathematics in Our Schools and its Contribution to War* by Sophia H. Levy,; also *Mathematics in the War Program* in *The Secondary Schools*, by Joseph B. Orleans. Both papers appeared in the November, 1943, issue of *The Mathematics Teacher.*

cally pointed out that the armed forces are *not* backing this additional attempt at forced feeding. They have demanded ***basic mathematics, not war applications.*** But not a few over-zealous persons, for whatever reason, are advocating what no average teacher or class of students can possibly accomplish within the allotted period of time. What are we to think, for example, of a prominent school system which proudly announces in the press that hereafter its junior high school program in mathematics is to include the elements of "logistics, ballistics, and aeronautics?" Surely, we must coöperate in every possible way in the direction of greater efficiency in the war effort. But we are only hindering that cause by violating all laws of sound pedagogic procedure, of organic growth, and of common sense.

The Organization of a Continuous Program in Elementary and Secondary Mathematics. Is it too much to hope that, after decades of endeavor, we shall at last have a ***continuous*** mathematical curriculum? For it should have become clear, from what has been said, that the traditional program does not, and never will, meet modern demands. What, then, is the outlook for a more integrated mathematical currilum? First, the Joint Commission Report of 1940 presents a six-year course in secondary mathematics. Second, some states and certain large school systems have already issued twelve-year programs in mathematics. Thus, in 1942, the State of North Carolina published its "Suggested Twelve Year Program" for all the basic subjects.[27] It now behooves all those who believe in the soundness of this movement to support it and to work for its success.

Two important considerations which regularly come up in discussions pertaining to a continuous curriculum have to do with the most favorable organization of school systems and with the problem of retarded or non-academic pupils. Naturally, reorganized schools following either the $6-6$ or the $6-4-4$ plan can more readily integrate their curricula than those based on the $8-4$ or the $6-3-3$ plan. Again, for non-college students of average ability, and for retarded pupils, the Joint Commission Report submits alternative programs. In general, these students should be given a broadly constructed course, covering at least two years beyond the eighth school year, or specialized courses needed in specific vocations. Naturally, during the war years, all pupils will have to strive first of all for literacy in the same basic fundamentals of mathematics.

Conclusion. We have presented a glimpse of the influences that are transforming the teaching of mathematics. On that basis, we have

[27] The school system of Washington, D. C., recently made available in tentative form a twelve-year program in mathematics.

outlined what seem to be the "next steps" during the years that lie ahead. Underlying this story there are certain assumptions and convictions. The universe in which we live appears to be based on the concepts of *order, structure,* and *relations.* By discovering and correctly applying its laws, we can overcome fear, solve many of our problems more easily, and face the future with greater confidence. The present world conflict is in large measure concerned with an extension of the concepts of order and law to *human* affairs. In these great enterprises, *mathematical thinking* and scientific training play a major role. For, in the language of Professor Hotelling, "there is no surer key to unlock all sorts of doors than mathematics."

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Brief Notes and Comments

Edited by
H. A. Simmons

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8. *Plane Right Triangles with Sides and Area in Arithmetic Progression.* On page 70 of the November, 1943, issue of this Magazine appears a statement, attributed to Lebesgue, that the plane right triangle with sides equal to 3, 4, 5 and area equal to 6 is the *only* plane right triangle whose sides and area form an arithmetic progression. Possibly Lebesgue's discussion was restricted to solutions in integers, although this is not stated; otherwise, there are three other such triangles.

Obviously the triangle cannot be isosceles, since a, a, $a\sqrt{2}$ cannot be in arithmetic progression.

Let a be the shorter leg, b the longer leg. Then the hypotenuse is $\sqrt{a^2+b^2}$, the area is $ab/2$, and $0<a<b<\sqrt{a^2+b^2}$. There are four possible solutions, depending on the relative magnitude of $ab/2$.

I. Let $0<a<b<\sqrt{a^2+b^2}<ab/2$.
Then $2b=a+\sqrt{a^2+b^2}$ and $2\sqrt{a^2+b^2}=b+ab/2$, whence $a=3$, $b=4$, $\sqrt{a^2+b^2}=5$, $ab/2=6$, the Lesbesgue result.

II. Let $0<a<b<ab/2<\sqrt{a^2+b^2}$.
Then $2b=a+ab/2$ and $ab=b+\sqrt{a^2+b^2}$, whence $a=4/3\sqrt{3}$, $b=1+\sqrt{3}$, $ab/2=2+(2/3)\sqrt{3}$, $\sqrt{a^2+b^2}=3+(1/3)\sqrt{3}$.

III. Let $0<a<ab/2<b<\sqrt{a^2+b^2}$.
Then $ab=a+b$ and $2b=ab/2+\sqrt{a^2+b^2}$, whence $a=(10/15\sqrt{6}$, $ab/2=(12+8\sqrt{6}/15)$, $b=(24+6\sqrt{6}/15)$, $\sqrt{a^2+b^2}=(36+4\sqrt{6}/15)$.

IV. Let $0<ab/2<a<b<\sqrt{a^2+b^2}$.
Then $2a=ab/2+b$ and $2b=a+\sqrt{a^2+b^2}$, whence $ab/2=2/3$, $a=(3/3)$, $b=(4/3)$, $\sqrt{a^2+b^2}=5/3$, a simple solution in rational fractions.

U. S. Military Academy Harris Jones.

Bibliography and Reviews

Edited by
H. A. SIMMONS and P. K. SMITH

Commercial Algebra. By Hugh E. Stelson and Harold P. Rogers. Macmillan Company, New York, 1943. xi+283 pages (including 54 pages of tables). $2.50.

This book contains 209 pages of exposition amd exercises, about three-fourths of which are devoted to customary topics of college algebra. In this usual material, the problems possess considerable newness, which should be refreshing to teachers. For example, on page 20, we find *Recipe Problems* in connection with proportion, and these are followed by *Chemical Problems, Pulley Problems,* etc., in such a way as to emphasize utility in a beautiful manner.

The novelty of the text, from the point of view of topics treated, is due mainly to its including chapters with the following titles: *Buying and Selling Merchandise; Simple Discount; Logarithms and Slide Rule; Consumer Credit.* The problems in these chapters are on the whole extremely interesting, and their solution involves many formulas that do not appear in most of the current textbooks on college algebra.

We are not certain that we agree with the authors as to the meanings of a few terms. One of these is *gambling*. The first sentence of Article 129 is this: "Gambling is the act of exchanging something small and certain for something large and uncertain". If the exchange can be made and if the *something* that is large and uncertain is more valuable than is the *something* that is small and certain, then gambling is profitable,—that is, if we accept the stated definition. However, if one reads all of Article 129, he is apt to be impressed with it and to consider it as splendid advice to students.

Although this book is written primarily for students who need commercial applications, it would be good for Liberal Arts students in general. Formal drill in manipulations with *fractions, radicals, exponents, linear equations, quadratic equations, progressions,* the *binomial theorem, permutations and combinations, probability,* and *annuities* can be obtained with an unusual degree of novelty from this text.

We should welcome an opportunity to use this text.

Northwestern University. H. A. SIMMONS.

Plane Trigonometry with Tables. By D. H. Ballou and F. H. Steen. Ginn and Co., Boston, 1943. vi+120 pages. $2.00.

This is a brief treatment of material for the conventional minimum course in plane trigonometry. The preface includes the statement that "the aim has been to develop the theory and to illustrate its applications as clearly as possible." It is the opinion of the reviewer that the authors have done as good a job of *developing the theory* as have most writers and that they have *illustrated its applications* most clearly and effectively. The exceptionally good figures (including twenty two scenic illustrations) and the wide choice of practical and timely problem material are outstanding features of the work.

The book is divided into eight chapters, the last of which is on logarithms. Radian and mil measures are introduced in the first chapter. The trigonometric functions of the general angle are defined first. No treatment of graphs of the functions is included. The remaining topics are found in the order given in most texts.

The material is well arranged and carefully written. The workmanship and printing are of the best. No typograhpical errors were observed. Only a few questionable points were noted. The expression *trigonometric function* was used four paragraphs before the term *function* was defined. The symbol $\pm$ did not receive the same careful attention in connection with the equation $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ that it did in other portions of the book.

Eighty four pages of tables, mostly five-place, include a table of natural haversines and their logarithms. The haversine is not used in the text, except for mention in a footnote.

Louisiana State University. F. A. RICKEY.

Analytic Geometry. By Frederick H. Steen and Donald H. Ballou. Ginn and Co., Boston, 1943.

This is a well written textbook which includes, with a few minor exceptions, all of the classic material of analytic geometry as well as many interesting features which are not encountered in the usual treatise upon this subject. A new treatment is employed in dealing with the matter of division of a given line segment into a specified ratio which is somewhat simpler and more direct than the usual approach. In the matter of the normal form of the equation of a line, a novel method of attack is used which seems to the reviewer to complicate, rather than simplify, the problem. However, it is certainly refreshing to find a book which does not follow the usual cut and dried method. The problem of rotation is also dealt with from a new point of view, and the system used involves less complicated algebraic manipulations than those required by the standard formulas. However, the authors have included the classical treatment of this problem so that the teacher can use it if he so desires. Another unusual feature is the blanket assumption that all lines are directed upward or, if horizontal, to the right. This seems at first thought to be a rather restrictive and unnecessary hypothesis, but there are no essential difficulties encountered because of it. The interpretation of the algebraic sign of the distance between a point and a line is different from the ordinary case. However, since both interpretations are arbitrary, there is no contradiction.

The book is well set up typographically, and the profuse illustrations are well done. The problems seem to be carefully chosen, and include many practical situations not ordinarily included in a book of this type. Another interesting and valuable feature is the inclusion of a set of "Orals" which precede nearly every set of exercises. These "Orals" will be welcomed by every teacher for the purposes of class room drill. In addition, several sections are devoted to additional points of interest, which include, for example, certain properties of the ellipse, parabola, and hyperbola.

The chapter on polar coordinates is very thorough, as is the one which deals with higher plane curves. The chapter on parametric equations describes several common curves including the cycloid, hypocycloid and epicycloid. The work upon the conics sections is very thorough and comprehensive. The matter of systems of lines is touched on only briefly, while systems of circles and their radical axes are not mentioned at all. The last four chapters are devoted to solid analytic geometry, and are sufficiently complete to give a student the fundamental ideas of this subject.

The single objectionable feature of the book is the prolific use of footnotes. In the 195 pages of text there are no less than 69 footnotes, with as many as three occurring on the same page. Most of these could have been included in the text without much difficulty. As it is, constant reference to footnotes destroys the continuity for the reader.

University of New Mexico. CHAS. B. BARKER.

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